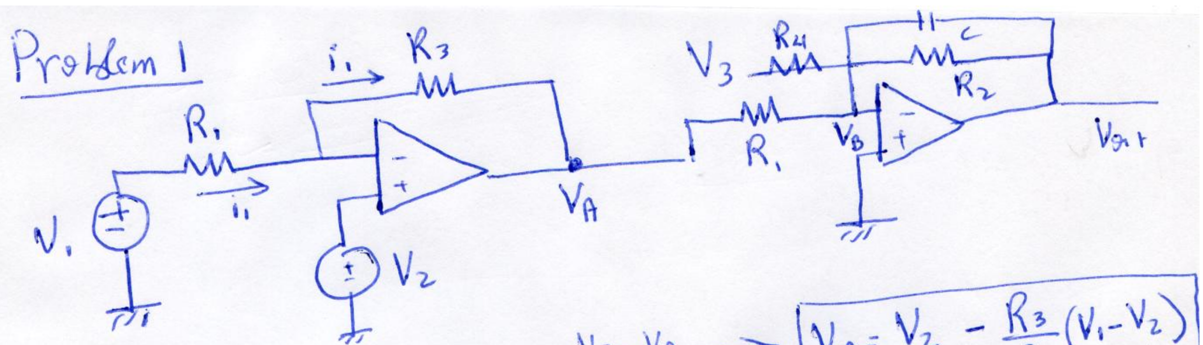
Problem 1



* USE KCL $\quad i_1 = \dfrac{V_1 - V_2}{R_1} = \dfrac{V_2 - V_A}{R_3} \Rightarrow$ $\boxed{V_A = V_2 - \frac{R_3}{R_1}(V_1 - V_2)}$

* In second stage, we have inverting amp ~~with $Z_1 = R_1$ and [illegible] $\frac{1}{sC}$~~

$$KCL \Rightarrow \frac{V_A - V_B}{R_1} + \frac{V_3 - V_B}{R_4} = \frac{V_B - V_{out}}{Z} \qquad \left(\begin{array}{l} Z = R_2 // (\frac{1}{sC}) \\ V_B = 0 \end{array} \right)$$

$$\Rightarrow \frac{V_A}{R_1} + \frac{V_3}{R_4} = \frac{-V_{out}}{\left(\frac{R_2}{R_2 sC + 1}\right)}$$

$$\Rightarrow \boxed{V_{out} = \frac{-R_2}{R_1 R_2 sC + R_1} V_A - \frac{R_2}{R_2 R_4 sC + R_4} V_3} \quad (2)$$

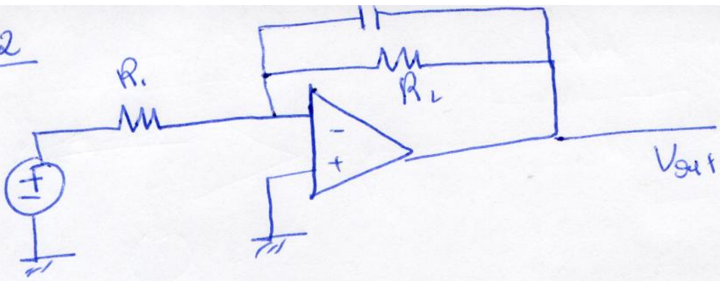
Combine (1) and (2)

$$V_{out} = \frac{-R_2}{R_1 R_2 sC + R_1} V_2 + \frac{R_2 R_3}{R_1^2 R_2 sC + R_1^2}(V_1 - V_2) - \frac{R_2}{R_2 R_4 sC + R_4} V_3$$

$$V_{out} = \frac{R_2 R_3}{R_1^2 R_2 sC + R_1^2} V_1 - \left(\frac{R_2}{R_1 R_2 sC + R_1} + \frac{R_2 R_3}{R_1^2 R_2 sC + R_1^2} \right) V_2 - \frac{R_2}{R_2 R_4 sC + R_4} V_3$$

$$\boxed{V_{out} = \left(\frac{R_2 R_3}{R_1^2 R_2 sC + R_1^2} \right) V_1 - \left(\frac{R_1 R_2 + R_2 R_3}{R_1^2 R_2 sC + R_1^2} \right) V_2 - \left(\frac{R_2}{R_2 R_4 sC + R_4} \right) V_3}$$

Problem 2



From Lecture 12 page 17

$$T(s) = -\left(\frac{R_2}{R_1}\right) \cdot \frac{1}{1 + sCR_2}$$

$R_2 = 10 \times 10^3 \Omega$, $R_1 = 2 \times 10^3$, $C = 100 \times 10^{-9} F = 10^{-7} F$

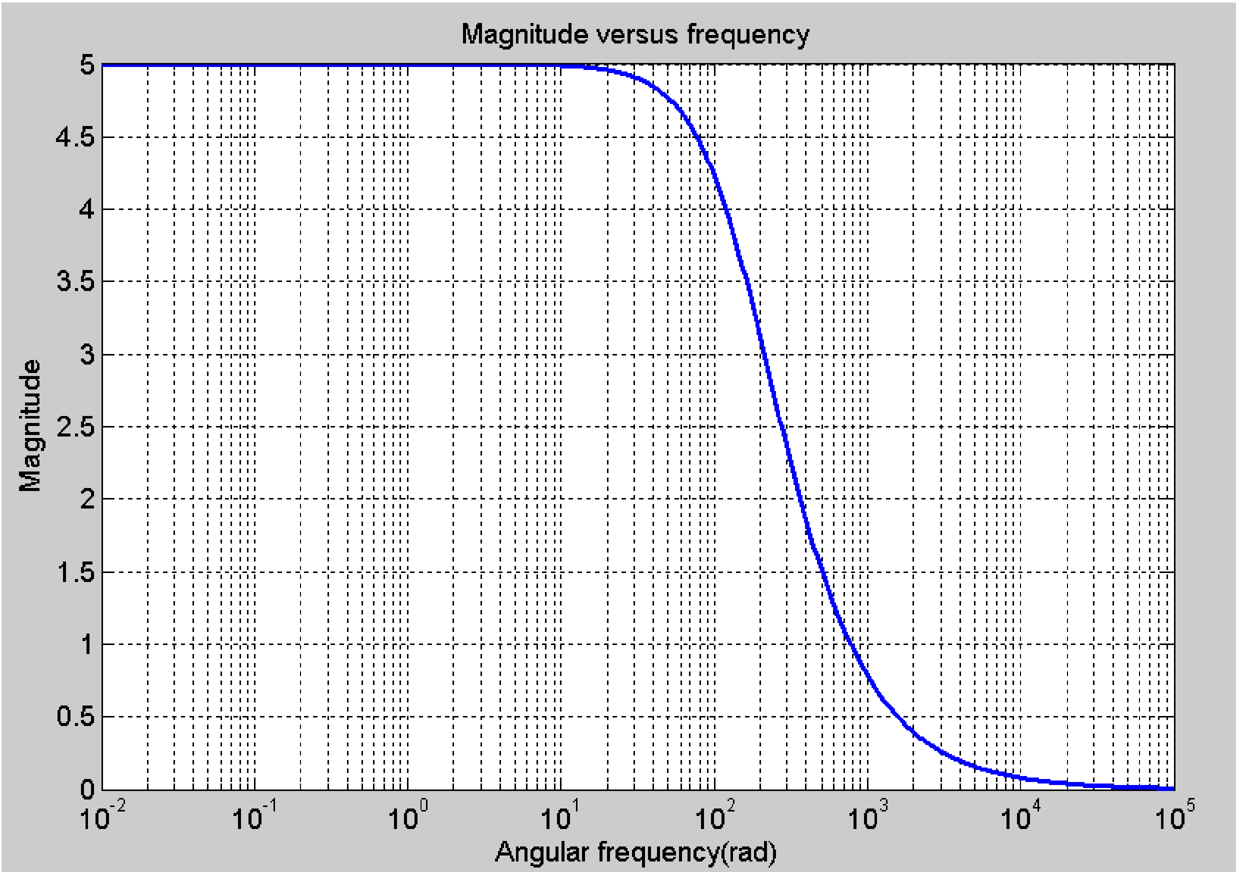
$$T(s) = -5 \cdot \frac{1}{1 + s \times 10^{-3}} = \frac{-5000}{s + 1000}$$

Pole at $s = -1000 \Rightarrow \omega_0 = -p = 1000 \text{ rad/s}$

$$|T(s)|_{\omega=\omega_0} = \frac{5000}{\sqrt{\omega^2 + 10^6}}\Bigg|_{\omega=1000} = \frac{5000}{\sqrt{2 \times 10^6}} = \frac{5000}{\sqrt{2} \times 10^3} = \frac{5}{\sqrt{2}}$$

Matlab code

```
clear all
close all
f= -2:.01:5;
f=10.^f;
w=2*pi*f;
T = 5000./sqrt(w.^2 + 10^6);
semilogx(f,T,'linewidth',2.5)
grid on
xlabel('Frequency(Hz)','fontsize',14),ylabel('Magnitude','fontsize',14)
set(get(gcf,'CurrentAxes'),'FontSize',14)
title('Magnitude versus frequency','fontsize',14)
```


Magnitude versus frequency
Magnitude
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
Angular frequency(rad)

Problem 3

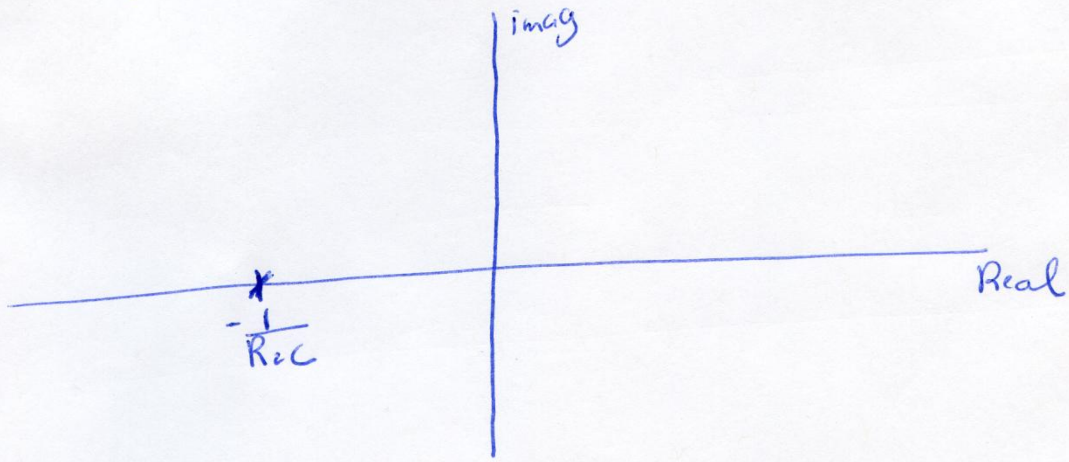
Similar to problem (2)

$$T(s) = -\left(\frac{R_2}{R_1}\right) \cdot \frac{1}{1 + sCR_2}$$

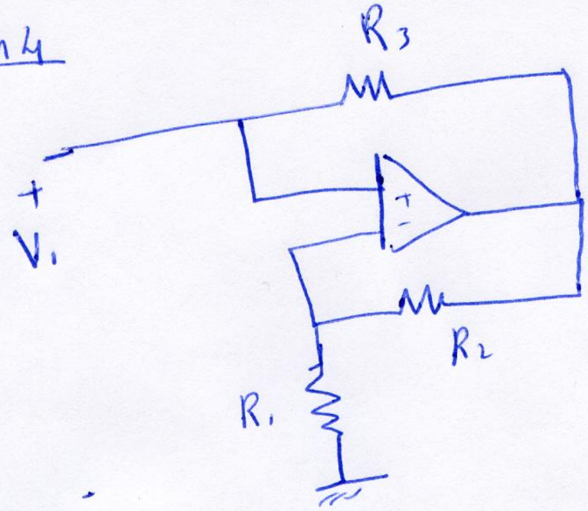
Given $A(s) = \dfrac{A_0}{\frac{s}{p} + 1}$ $\Rightarrow$ $-\dfrac{R_2}{R_1} = A_0$ and $CR_2 = \dfrac{1}{p}$

or $\boxed{A_0 = -\dfrac{R_2}{R_1}, \quad p = \dfrac{1}{R_2 C}}$

Pole at $s = -\dfrac{1}{R_2 C}$



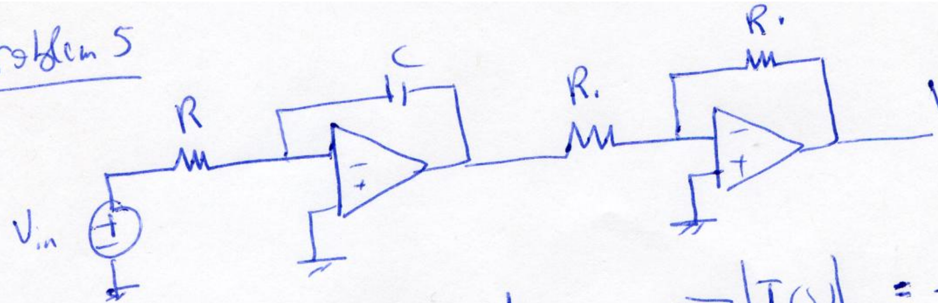
Problem 4



from Lecture 15 pg 20 $\quad Z_{in} = -\frac{R_1 R_3}{R_2}$

Choose $R_1 = 2k\Omega \quad R_2 = 1k\Omega \quad R_3 = 10k\Omega$

Problem 5



$$T(s) = \frac{1}{sRC} \Rightarrow |T(s)| = \frac{1}{\omega RC}$$

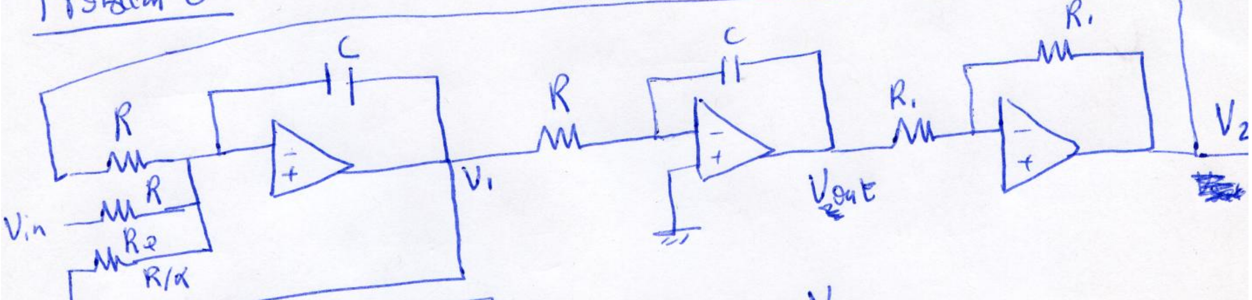
$$f = 10\text{ KHz} \Rightarrow |T(s)| = 1 \Rightarrow \omega = \frac{1}{RC}$$

or $$\boxed{RC = \frac{1}{\omega} = \frac{1}{2\pi \times 10^4}}$$

Choose $\boxed{C = 10\,nF}$ $\Rightarrow R = \frac{1}{2\pi\times10^4}\cdot\frac{1}{10\times10^{-9}} = \frac{1}{2\pi}\times10^4$

$$\boxed{R = 1.59\ K\Omega}$$

Problem 6



$$\boxed{V_{out} = -V_1\left(\frac{1}{sRC}\right)} \quad (1) \qquad V_{out} = -V_2$$

$$V_1 = \left(\frac{-1}{sRC}\right)V_2 + \left(\frac{-1}{sRC}\right)V_{in} + \left(\frac{-1}{sR_QC}\right)V_1$$

$$\Rightarrow \boxed{V_1\left(1 + \frac{1}{sR_QC}\right) = \frac{V_{out}}{sRC} - \frac{1}{sRC}V_{in}} \quad (2)$$

Replace (2) in (1)

$$V_{out} = \left(\frac{-1}{sRC}\right)\frac{sR_QC}{1+sR_QC}\left(\frac{V_{out} - V_{in}}{sRC}\right)$$

$$V_{out} = \frac{-R_Q}{R + sRR_QC}\left(\frac{V_{out} - V_{in}}{sRC}\right)$$

$$V_{out}\left(1 + \frac{R_Q}{sR^2C + s^2R^2R_QC^2}\right) = \frac{R_Q}{R^2sC + s^2R^2R_QC^2}V_{in}$$

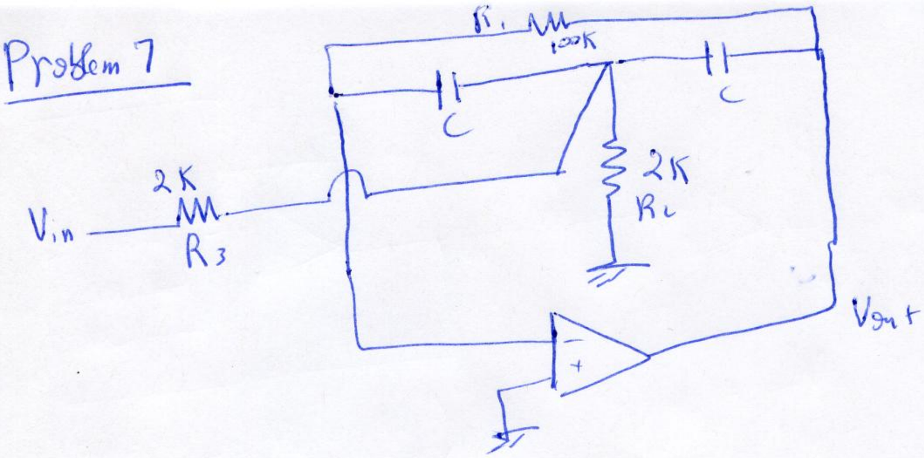
$$\frac{V_{out}}{V_{in}} = \frac{R_Q}{R^2sC + s^2R^2R_QC^2} \times \frac{sR^2C + s^2R^2R_QC^2}{R_Q + sR^2C + s^2R^2R_QC^2}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{R_Q}{(R^2R_QC^2)s^2 + (R^2C)s + R_Q} = \frac{\left(\frac{1}{RC}\right)^2}{s^2 + s\left(\frac{1}{R_QC}\right) + \left(\frac{1}{RC}\right)^2}}$$

$$BW = \frac{1}{R_QC} = \boxed{\frac{\alpha}{RC}} \qquad \omega_p = \sqrt{\left(\frac{1}{RC}\right)^2} = \boxed{\frac{1}{RC}}$$

$\boxed{\text{No Zeros}}$ poles (1) $\boxed{s = \dfrac{-\frac{\alpha}{RC} \pm \sqrt{\frac{\alpha^2 - 4}{(RC)^2}}}{2}}$

Problem 7



Lecture (12) pg 32

$$T(s) = -\frac{\frac{s}{R_3 C}}{s^2 + s\left(\frac{2}{R_1 C}\right) + \frac{1}{(R_2 // R_3) R_1 C^2}}$$

$$\omega_p = \frac{1}{\sqrt{R_1 (R_2 // R_3)}\, C} \qquad BW = \frac{2}{R_1 C} \qquad K = \frac{R_1}{2R_3}$$

$BW = \frac{2}{R_1 C} = 2000 \text{ rad/s}$ $\qquad \omega_p = 10{,}000 \text{ rad/s}$

$K = 25$

$\omega_L = \omega_p - \frac{BW}{2} = 9000 \text{ rad/s}$

$\omega_H = \omega_p + \frac{BW}{2} = 11{,}000 \text{ rad/s}$

Poles @ $-9{,}000$ rad/s and $-11{,}000$ rad/s

Problem 8

Use circuit from problem 7

$BW = 500\,Hz$ or $BW = 1000\pi$ rad/s

$\omega_p = 5\,kHz$ or $\omega_p = 10{,}000\pi$ rad/s

let $C = 10\,nF$ $\Rightarrow$ $\boxed{R_1 = \frac{2}{BW \cdot C} = 63.7\,k\Omega}$

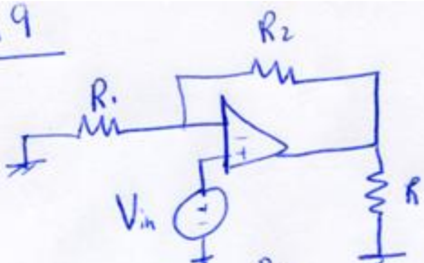
let $K = 10$ $\Rightarrow$ $\boxed{R_3 = \frac{R_1}{2K} = 3.18\,k\Omega}$

$$\omega_p = 10{,}000\pi = \frac{1}{C}\sqrt{\frac{1}{R_1}\left(\frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$\Rightarrow$ $\boxed{R_2 = 167.5\,\Omega}$

Problem 9

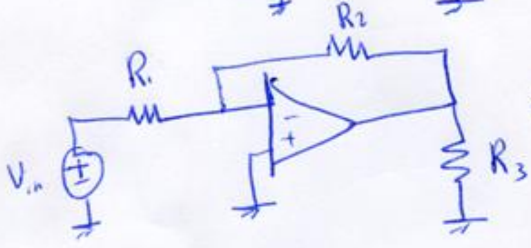
a)



$\Rightarrow$ Noninverting amp

$BW = \frac{GB}{K_o} = \frac{2\pi\,MHz}{1 + \frac{R_2}{R_1}} = \boxed{400\,kHz}$

b)



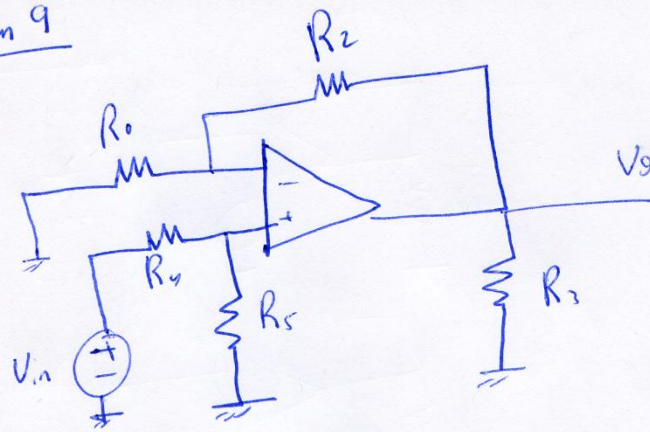
$\Rightarrow$ Inverting amp

$BW = \frac{GB}{1 + |K_o|} = \frac{2\pi\,MHz}{1+8} = \boxed{222\,k}$

V+

Problem 9

c)



$$V^- = \frac{V_{out} R_1}{R_1 + R_2} = \frac{V_{out}}{9} \Rightarrow V_{out} = (V^+ - V^-) A_o(s)$$

$$\boxed{V_{out} = \frac{GB}{s}\left(\frac{V_{in}}{2} - \frac{V_{out}}{9}\right)} \quad (1)$$

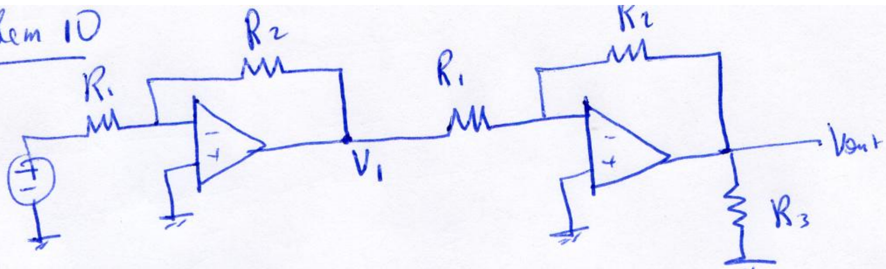
$$V^+ = \frac{V_{in} R_5}{R_4 + R_5} = \frac{V_{in}}{2}$$

from (1)

$$\frac{V_{out}}{V_{in}} = \frac{9/2}{1 + \frac{9s}{GB}} \Rightarrow BW = \frac{GB}{9}$$

$$\boxed{BW = \frac{2MHz}{9} = 222\ KHz}$$

Problem 10



1st stage

$$V_1 = \frac{GB}{s}(V^+ - V^-) = -\frac{GB}{s} \cdot V^- = -\frac{GB}{s}\left(V_{in} + \frac{R_1(V_1 - V_{in})}{R_1 + R_2}\right)$$

or $$V_1 = -\frac{GB}{s}\left(V_{in} + \frac{1}{5}V_1 - \frac{1}{5}V_{in}\right)$$

or $$V_1\left(1 + \frac{GB}{5s}\right) = -\frac{4}{5s} GB \, V_{in}$$

or $$\boxed{V_1 = \frac{-4\,GB}{5s + GB} V_{in}}$$

$$\Rightarrow \boxed{BW = \frac{GB}{1+K_0} = \frac{750}{1+4} = 150KH}$$

2nd stage

$$\boxed{V_{out} = \frac{-4\,GB}{5s + GB} V_1}$$

$$\boxed{V_{out} = \frac{16\,GB^2}{25s^2 + 10GBs + GB^2} V_{in}}$$

$$\boxed{|T(j\omega)| = \frac{16GB^2}{|(5s+GB)^2|} = \frac{16\,GB^2}{25\omega^2 + GB^2}}$$

$\omega = 0 \Rightarrow |T(j\omega)| = 16$

$\Rightarrow \boxed{K_0 = 16}$

$$|T(j\omega)| = \frac{16}{\sqrt{2}} \Rightarrow 16\sqrt{2}\,GB^2 = 16(25\omega_B^2 + GB^2) \Rightarrow \omega^2 = GB^2\frac{(\sqrt{2}-1)}{25}$$

$$\Rightarrow \boxed{\omega_B = GB\sqrt{\frac{\sqrt{2}-1}{25}} = 606.57\ rad/s} = BW$$

$$\boxed{f_B = 96.5\ KHz}$$

$$\boxed{~~BW = \frac{GB}{K_0} = \frac{750KHz}{16} = 46.87\ KHz~~}$$

~~single stage~~ →

For a 1st stage amplifer with gain 16, we have

$$BW = \frac{GB}{K_0} = \frac{750\,KHZ}{16} = \boxed{46\,KHZ}$$

Therefore, there is a significant improvement when we cascade 2 stages.